How Well Do We Know the High-Density Equation of State?

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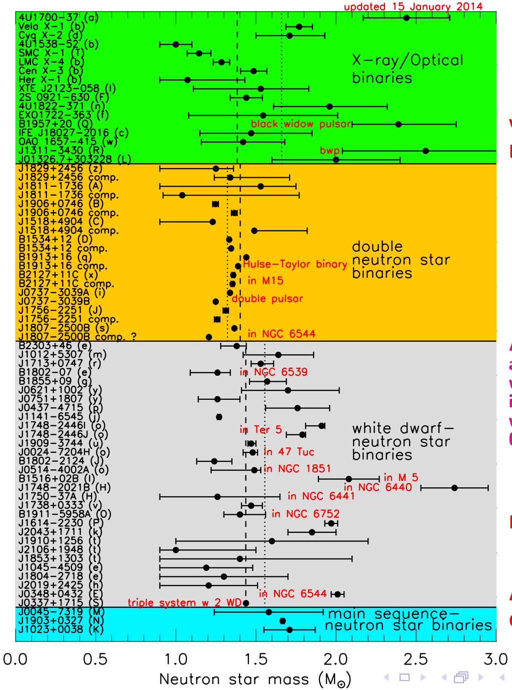
Outline

- General Constraints on Neutron Star Structure From Mass Measurements
- The Neutron Star Radius and the Nuclear Symmetry Energy
- Nuclear Experimental Constraints on the Symmetry Energy
- Constraints from Pure Neutron Matter Theory
- Astrophysical Constraints
  - Pulsar and X-ray Binary Mass Measurements
  - Photospheric Radius Expansion Bursts
  - Thermal Emission from Isolated and Quiescent Binary Sources
  - Other Proposed Mass and Radius Constraints



vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is 0.23 $M_\odot$ larger, weighted average is 0.04 $M_\odot$ smaller

Demorest et al. 2010

Antoniadis et al. 2013

Champion et al. 2008

Causality + GR Limits and the Maximum Mass

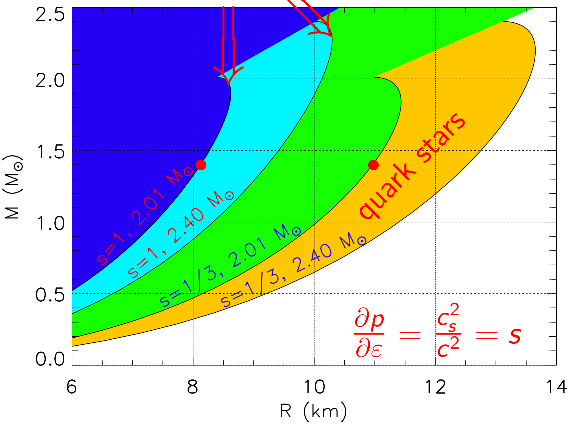
A lower limit to the maximum mass sets a lower limit to the radius for a given mass.

Similarly, a precise (M, R) measurement sets an upper limit to the maximum mass.

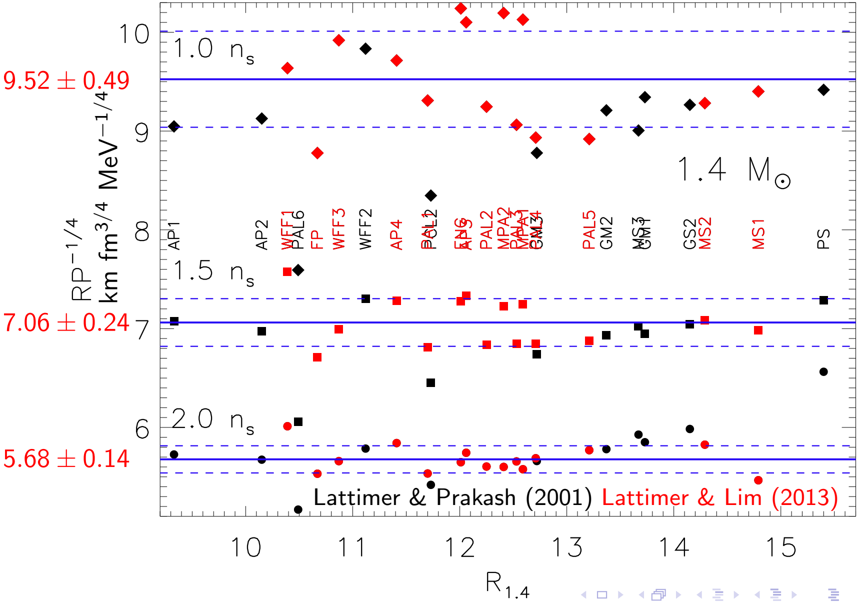
$1.4 M_\odot$ stars must have $R > 8.15 M_\odot$.

$1.4 M_\odot$ strange quark matter stars (and likely hybrid quark/hadron stars) must have $R > 11$ km.

$M - R$ curves for maximally compact EOS



The Radius – Pressure Correlation



Nuclear Symmetry Energy

Defined as the difference between energies of pure neutron matter ($x = 0$) and symmetric ($x = 1/2$) nuclear matter.

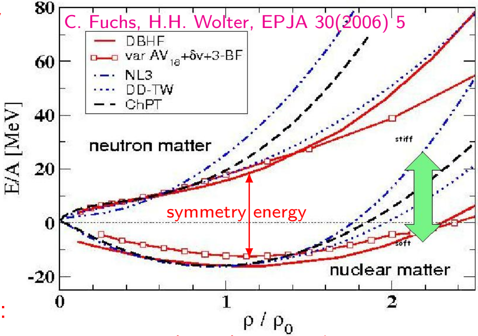
$$S(\rho) = E(\rho, x = 0) - E(\rho, x = 1/2)$$

Expanding around the saturation density (ρ_s) and symmetric matter ($x = 1/2$)

$$E(\rho, x) = E(\rho, 1/2) + (1 - 2x)^2 S_2(\rho) + \ldots$$

$$S_2(\rho) = \mathbf{S_v} + \frac{\mathbf{L}}{3}\frac{\rho - \rho_s}{\rho_s} + \ldots$$

$$\mathbf{S_v} \simeq 31 \text{ MeV}, \quad \mathbf{L} \simeq 50 \text{ MeV}$$



Connections to pure neutron matter:

$$E(\rho_s, 0) \approx S_v + E(\rho_s, 1/2) \equiv S_v - B, \qquad p(\rho_s, 0) = L\rho_s/3$$

Neutron star matter (in beta equilibrium):

$$\frac{\partial(E + E_e)}{\partial x} = 0, \quad p(\rho_s, x_\beta) \simeq \frac{L\rho_s}{3}\left[1 - \left(\frac{4S_v}{\hbar c}\right)^3 \frac{4 - 3S_v/L}{3\pi^2\rho_s}\right]$$

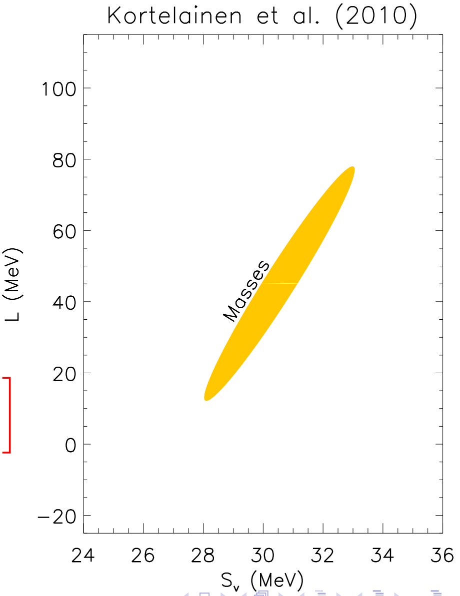
Nuclear Experimental Constraints

Binding Energies

Liquid Droplet Model

$$E_{sym} = AI^2\left[\frac{S_v}{1+S_sA^{-1/3}/S_v} - \frac{Ze^2}{20R}\frac{S_sA^{-1/3}/S_v}{1+S_sA^{-1/3}/S_v}\right]$$

$$\frac{S_s}{S_v} \simeq \frac{3a}{2r_o}\left[1 + \frac{L}{3S_v} + \left(\frac{L}{3S_v}\right)^2 \cdots\right]$$

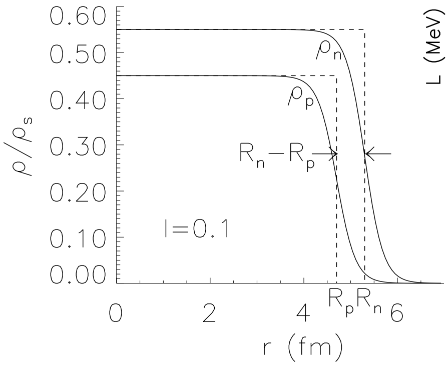


Nuclear Experimental Constraints

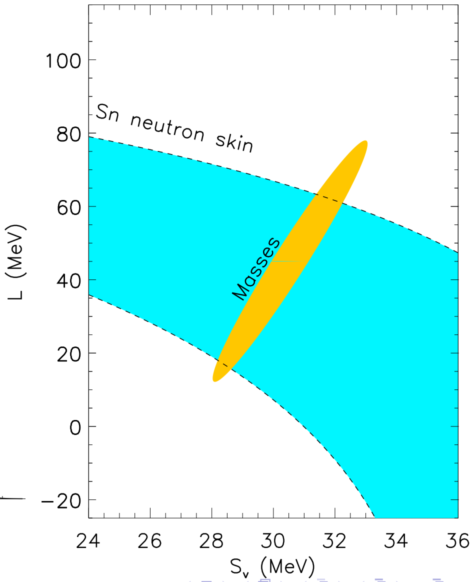
Neutron Skin Thicknesses

$$r_{np} = \frac{2r_o}{3S_v}\frac{1}{\sqrt{1-I^2}}\left(1+S_sA^{-1/3}/S_v\right)^{-1}$$
$$\times\sqrt{\frac{3}{5}}\left[IS_s - \frac{3Ze^2}{140r_o}\left(1+\frac{10}{3}\frac{S_sA^{-1/3}}{S_v}\right)\right]$$

$$r_{np,208} = 0.15 \pm 0.04 \text{ fm}$$

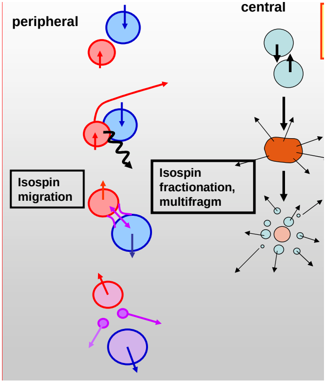


Tarbert et al. (2014)



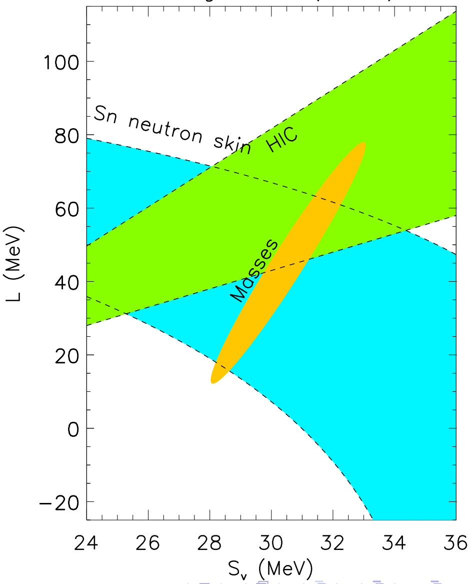
Nuclear Experimental Constraints

Flows in
Heavy Ion Collisions



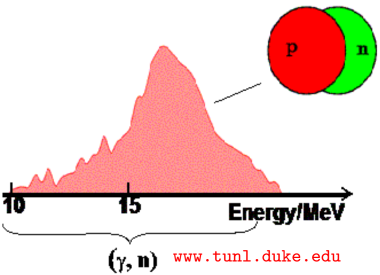
Wolter, NuSYM11

Tsang et al. (2009)



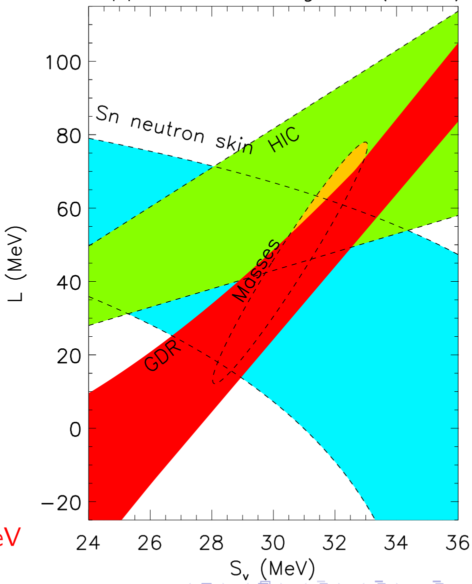
Nuclear Experimental Constraints

Giant Dipole Resonance Centroids



23.3 MeV$<S_2(0.1\ \mathrm{fm}^{-3})<$24.9 MeV

Trippa, Colo & Vigezzi (2008)



Nuclear Experimental Constraints

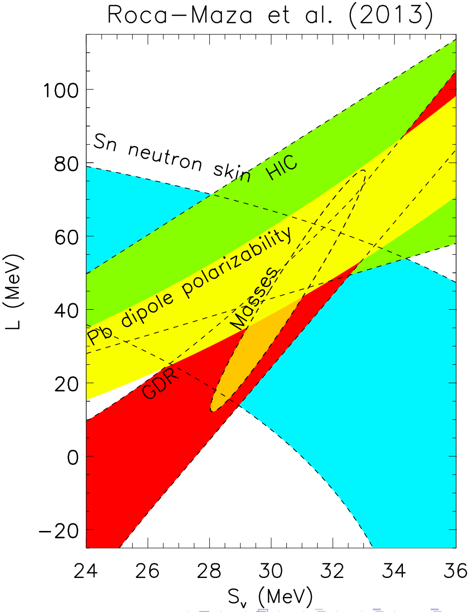
Dipole Polarizabilities

$\alpha_D = 4m_{-1}$

$\simeq \frac{AR^2}{20S_v}\left(1 + \frac{5}{3}\frac{S_s A^{-1/3}}{S_v}\right)$

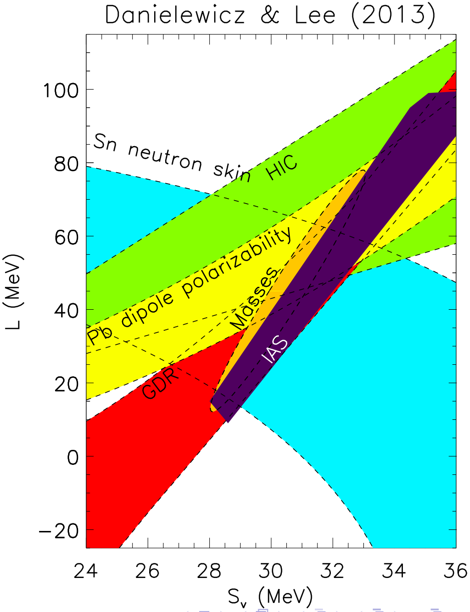
Uses data of
Tamii et al. (2011)

$\alpha_{D,208} = 20.1 \pm 0.6 \text{ fm}^2$



Nuclear Experimental Constraints

Isobaric Analog States



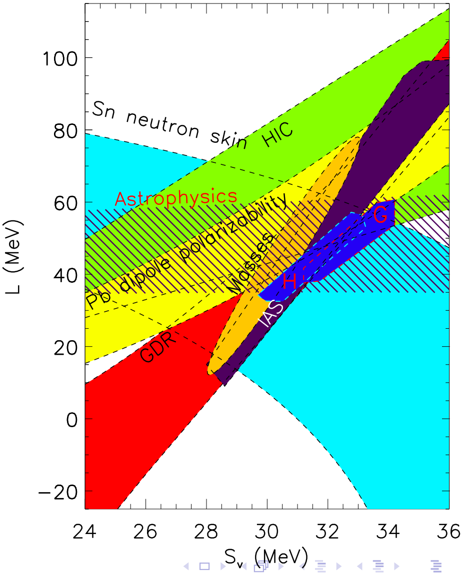
Theoretical Neutron Matter Calculations

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo

$S_v - L$ constraints from Hebeler et al. (2012)

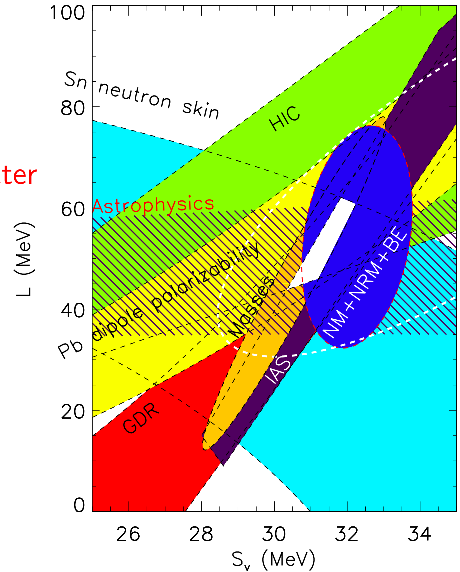
Gandolfi, Carlson & Reddy (2011);
Hebeler & Schwenk (2011)



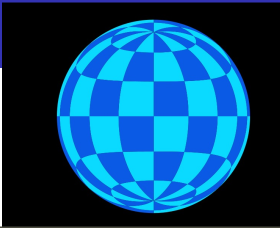
Theoretical Neutron-Rich Matter Calculations

Chiral Lagrangian studies of neutron and neutron-rich matter by Drischler, Somá & Schwenk (2014)

Includes uncertainties in symmetric matter properties



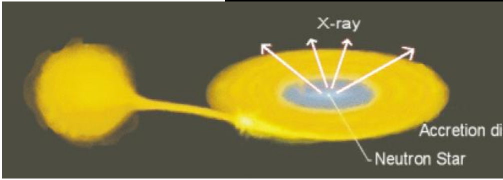
Simultaneous Mass/Radius Measurements



- Measurements of flux $F_\infty = (R_\infty/D)^2 \sigma T_{\rm eff}^4$ and color temperature $T_c \propto \lambda_{\rm max}^{-1}$ yield an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance D, interstellar absorption N_H, atmospheric composition

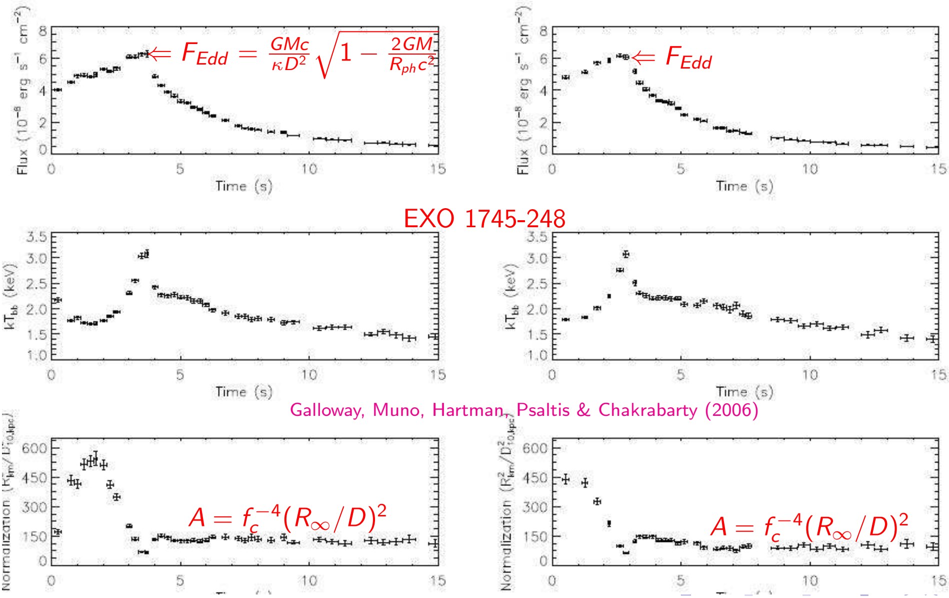


Best chances for accurate radius measurement:

- Nearby isolated neutron stars with parallax (uncertain atmosphere)
- Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters (reliable distances, low B H-atmosperes)
- Bursting sources (XRBs) with peak fluxes close to Eddington limit (where gravity balances radiation pressure)

$$F_{\rm Edd} = \frac{cGM}{\kappa D^2}\sqrt{1-2GM/Rc^2}$$

Photospheric Radius Expansion X-Ray Bursts



PRE Burst Models

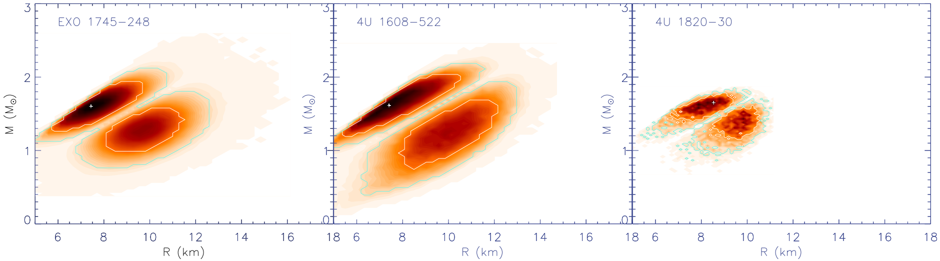
Ozel et al. $z_{\rm ph} = z$ $\beta = GM/Rc^2$ Steiner et al. $z_{\rm ph} << z$

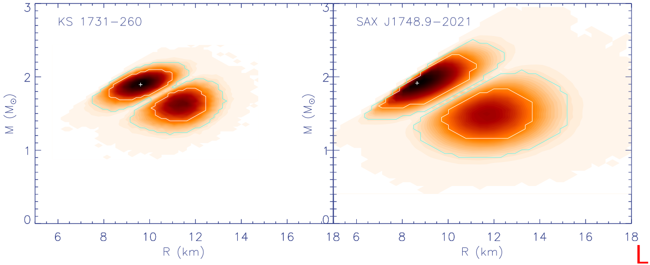
$$
\begin{aligned}
F_{\rm Edd,\infty} &= \frac{GMc}{\kappa D}\sqrt{1-2\beta} \\
A &= \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2 \\
\alpha &= \frac{F_{\rm Edd,\infty}}{\sqrt{A}}\frac{\kappa D}{f_c^2 c^3} = \beta(1-2\beta) \\
\gamma &= \frac{A f_c^4 c^3}{\kappa F_{\rm Edd,\infty}} = \frac{R_\infty}{\alpha} \\
\beta &= \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha} \\
\alpha &\le \frac{1}{8} \text{ required.}
\end{aligned}
$$

$$
\begin{aligned}
F_{\rm Edd,\infty} &= \frac{GMc}{\kappa D} \\
\alpha &= \beta\sqrt{1-2\beta} \\
\theta &= \cos^{-1}\left(1-54\alpha^2\right) \\
\beta &= \frac{1}{6}\left[1+\sqrt{3}\sin\left(\frac{\theta}{3}\right)\right. \\
&\qquad \left. -\cos\left(\frac{\theta}{3}\right)\right] \\
\alpha &\le \sqrt{\frac{1}{27}} \simeq 0.192 \text{ required.}
\end{aligned}
$$

	α			
EXO 1745-248	4U 1608-522	4U 1820-30	KS 1731-260	SAX J1748.9-2021
0.188 ± 0.035	0.247 ± 0.058	0.235 ± 0.04	0.199 ± 0.032	0.177 ± 0.036

$M - R$ PRE Burst Estimates



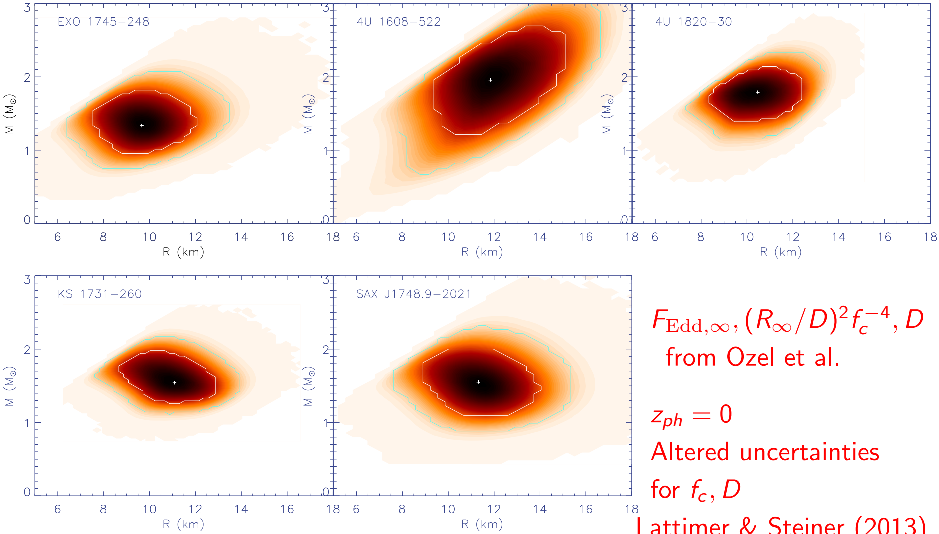


$F_{\mathrm{Edd},\infty}$, $(R_\infty/D)^2 f_c^{-4}$, D, f_c from Ozel et al.

$z_{ph} = z$

Lattimer & Steiner (2013)

$M - R$ PRE Burst Estimates



$F_{\mathrm{Edd},\infty}, (R_\infty/D)^2 f_c^{-4}, D$ from Ozel et al.

$z_{ph} = 0$

Altered uncertainties for f_c, D

Lattimer & Steiner (2013)

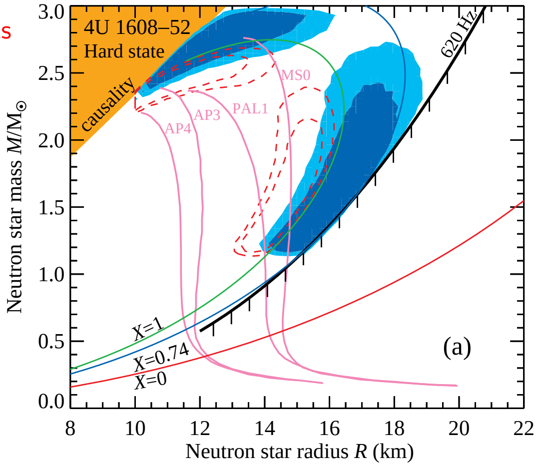
PRE Burst Conundrum

Poutanen et al. (2014) and Suleimanov et al. (2011) argue that soft short Type I bursts are affected by accretion discs that obscure our view.

This leads to underestimates of $F_{\mathrm{Edd},\infty}$ and F_∞.

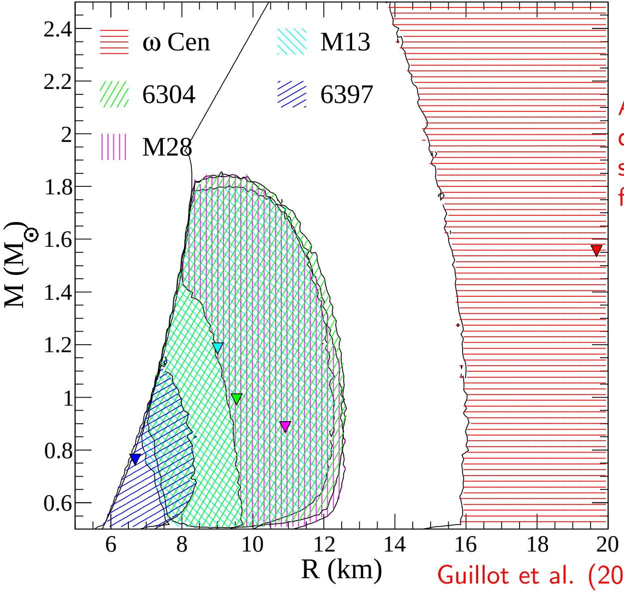
They also claim that f_c should be about 1.2 times larger.

Thus, estmates of α would remain roughly unchanged, but those of γ would be larger by f_c^4, leading to increases in radius estimates by the same factor.



They claim hard longer bursts should instead be used to infer masses and radii.

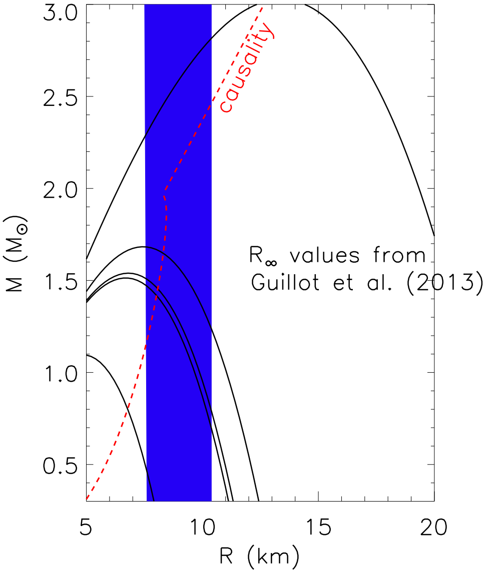
$M - R$ QLMXB Estimates



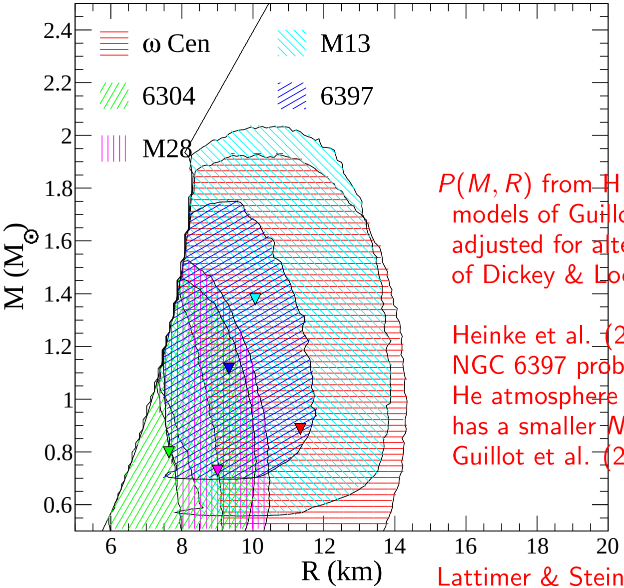
Absorption (N_H) determined self-consistently from spectra

Guillot et al. (2013)

Interpretation



$M - R$ QLMXB Estimates



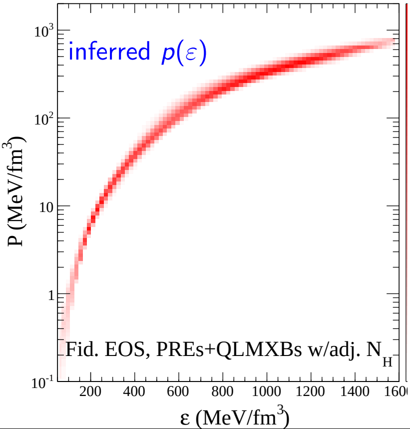
$P(M, R)$ from H atmosphere models of Guillot et al. (2013), adjusted for alternate N_H values of Dickey & Lockman (1990).

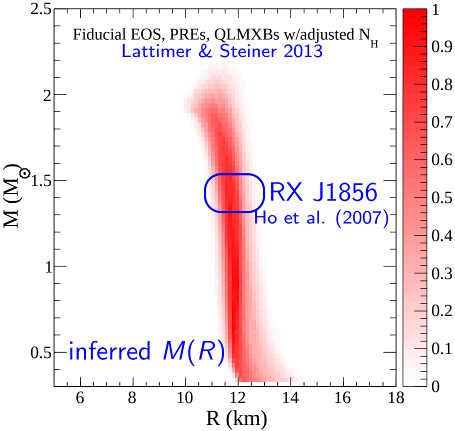
Heinke et al. (2014) found NGC 6397 probably has a He atmosphere and ω Cen has a smaller N_H than Guillot et al. (2013) found.

Lattimer & Steiner (2013)

Bayesian TOV Inversion

- $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- Polytropic EOS: $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1; $\varepsilon > \varepsilon_2$: n_2
- EOS parameters $K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$ uniformly distributed
- $M_{\rm max} \geq 1.97$ M$_\odot$, causality enforced
- All 10 stars equally weighted





Astronomy vs. Astronomy vs. Physics

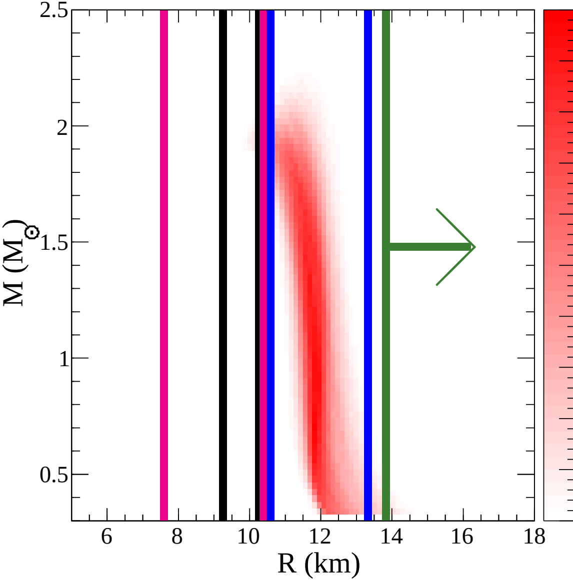
Ozel et al., PRE bursts $z_{\rm ph}$
z: $R = 9.74 \pm 0.50$ km.

Suleimanov et al., long PRE bursts: $R_{1.4} \gtrsim 13.9$ km

Guillot et al. (2013), all stars have the same radius, self N_H: $R = 9.1^{+1.3}_{-1.5}$ km.

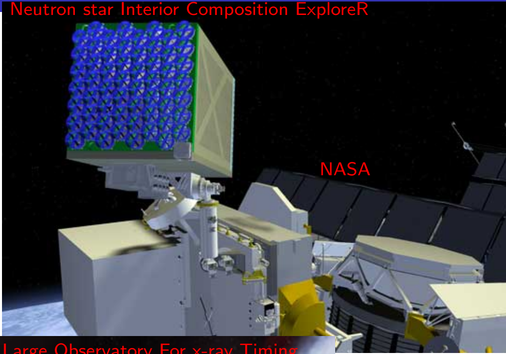
Lattimer & Steiner (2013), TOV, crust EOS, causality, maximum mass $> 2M_\odot$, $z_{\rm ph} = z$, alt N_H.

Lattimer & Lim (2013), nuclear experiments:
29 MeV $< S_v < 33$ MeV,
40 MeV $< L < 65$ MeV,
$R_{1.4} = 12.0 \pm 1.4$ km.



Additional Proposed Radius and Mass Constraints

- **Pulse profiles**
  Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling $\rightarrow M/R$; phase-resolved spectroscopy $\rightarrow R$.
- **Moment of inertia**
  Spin-orbit coupling of ultra-relativistic binary pulsars (e.g., PSR 0737+3039) vary i and contribute to $\dot{\omega}$: $I \propto MR^2$.
- **Supernova neutrinos**
  Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.
- **QPOs from accreting sources**
  ISCO and crustal oscillations



Constraints from Observations of Gravitational Radiation

Mergers:

Chirp mass $\mathcal{M} = (M_1 M_2)^{3/5} M^{-1/5}$ and tidal deformability $\lambda \propto R^5$ (Love number) are potentially measurable during inspiral.

$\bar{\lambda} \equiv \lambda M^{-5}$ is related to $\bar{I} \equiv I M^{-3}$ by an EOS-independent relation (Yagi & Yunes 2013). Both $\bar{\lambda}$ and $\bar{I}$ are also related to M/R in a relatively EOS-independent way (Lattimer & Lim 2013).

- Neutron star - neutron star: $M_{\rm crit}$ for prompt black hole formation, $f_{\rm peak}$ depends on R.
- Black hole - neutron star: $f_{\rm tidal\ disruption}$ depends on $R, a, M_{\rm BH}$. Disc mass depends on $a/M_{\rm BH}$ and on $M_{\rm NS} M_{\rm BH} R^{-2}$.

Rotating neutron stars: r-modes